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FACTOR ANALYSIS IN THE PROCESS OF DESIGNING OF COMPLEX OPTICAL SYSTEMS

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Abstract: *A model of factor analysis is developed in the process of designing of complex optical systems. Factor analysis is indispensible in designing of photometric and spectrophotometric optical-electronic devices which consist of entrance-scanning system by space, lens with inner focusing, collimator objective which ensure the entering of a parallel bundle of rays over a dispersing diffraction grate over a wave length, a chamber lens and respectively, a sensor.*

These and similar optical systems require higher degree of elimination of chromatic aberration, etc. which ensures good quality of the optical system and of the obtained results.

ФАКТОРЕН АНАЛИЗ В ПРОЦЕСА НА ПРОЕКТИРАНЕ НА СЛОЖНИ ОПТИЧНИ СИСТЕМИ

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Резюме: *Разработен е модел на факторен анализ в процеса на проектиране на сложни оптични системи. Факторният анализ е особено необходим при проектиране на фотометрични спектрофотометрични оптико-електронни уреди, състоящи се примерно от входно-сканираща система по пространство, обектив с вътрешна фокусировка, колиматорен обектив осигуряващ постъпване на паралелен сноп лъчи върху диспергиращa дифракционна решетка по дължина на вълна, камерен обектив и съответно сензор.*

Именно тези и подобни оптични системи изискват висока степен на отстраняване на хроматичната аберация и др, което да осигури високо качество на оптичната система и на получаваните резултати.

The method factor analysis ensures a possibility for constructional definition of the mathematical model of the optic system, and in particular the modal of its basic components and a set of factors within the limits of which the given task and questions should be solved. The model factor analysis is developed on the basis of experimental data. One of the typical forms of presentation of the experimental data is the matrixes. The columns of such a matrix correspond to the characteristics or its changes, and the rows correspond to different versions of the system. They are distinct by a number of specific values of the initial parameters. Using data from such an ensemble for the system, equations are being solved which connect the characteristics with the construction parameters.

The design of the optical system is a complex creative process which consists of many stages, including aberrations analysis, which ensure the evaluation of the correction quality of the system and its correction possibilities; elimination of the aberrations to a certain extent; optimization of the criterion for balancing the aberrations and the correction quality for a large quantity shafts and rays; calculation of the limits of optical parameters, etc.

The calculation of an optical system with certain characteristics $\varphi(\vec{P})$ is led to a solution of a system of nonlinear equations

(1)
$$\varphi_j(\vec{P}) = \tilde{\varphi}_j .$$

where the characteristics φ_j in certain areas of the area P are expanded in a Tyler row by degree of transformation Δp_i and they are limited by linear (or quadratic) terms

(2)
$$\varphi_j = \varphi_j^{(0)} + \sum_{i=1}^{m} \frac{d\varphi_i}{dp_i} \Delta p_i , \; j = 1,2,3\ldots,t , \; j = 1,2,3\ldots,m$$

The necessary appointed values of the characteristics $\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots, \tilde{\varphi}_t$, are constructionally given and they should be achieved with a certain extent of accuracy $\delta_{\varphi_1}, \delta_{\varphi_2}, \ldots, \delta_{\varphi_t}$, and for every correctional parameter p_i - initial value $p_i^{(0)}$, the sum of which makes the vector $\vec{P}^{(0)}$. Looking for the solution of equation (1) is done by means of creating a sequence of vectors
$\vec{P}^{(0)}, \vec{P}^{(1)}, \vec{P}^{(2)}, \ldots, \vec{P}^{(n)}$
And

(3)
$$\left|\varphi_j\left(\vec{P}^{(n)} - \tilde{\varphi}_j\right)\right| < \delta\varphi_j ,$$

where $j = 1,2,\ldots,t$

Defining $\vec{P}^{(n)}$, and thus answering condition (3), system (1) is being solved.
When entering a helping sequence of the function

(4)
$$\varphi_i^* = a_j^* \left(\frac{\varphi_j - \tilde{\varphi}_j}{\delta\varphi_j} \right)^2 ,$$

So the proximity to the solution of (1) is evaluated by the following subordination:

$$\Phi^* = \left\|\varphi^*\right\| = \sum_{j=1}^{t} a_j^* \left(\frac{\varphi_j - \tilde{\varphi}_j}{\delta\varphi_j} \right)^2 ,$$

where the non-negative value of a_j^* takes into consideration the influence of the changes of the characteristics of the evaluating function Φ^*. The values of a_j^* and $\delta\varphi_j$ are given by the constructor according to his opinion. The difficulties here come from defining the proper coefficients $\frac{a_j^*}{\sigma_{\varphi j}^2}$ which set the requirements to a certain aberration image, described by a large number of characteristics.

The method of calculation allows for a diversion of sizes and characteristics of the parts and the system, which have not been developed. The matter of concrete limits is decided on the basis of analysis of the influence of the changes of the construction parameters over the system characteristics.

One of the methods which aids the solution of this extremely complex problem, is finding the degree of influence of the parameters within certain limits and their changes over the described combination in equation (2) of the optical system characteristics.

If the system has t characteristics $\varphi_1, \varphi_2, \ldots, \varphi_t$, every one of them contains N quantities ($j = 1,2,\ldots,k,t; k = 1,2,\ldots,N$). We mark the values $\frac{\varphi k_j - \overline{\varphi}_j}{\sigma\varphi_j}$ by means of $\hat{\varphi}_j$ where $\overline{\varphi}_j$ is an average arithmetical value of the quantities of φ_{kj}; $\delta_{\varphi j}$ is the value of the dispersion, characterizing the dispersion of the values of φ_{kj} in relation with $\overline{\varphi}_j$. Then in the analysis of the main components, the basic equation is:

(5)
$$f^{(P)} = \sum_{j=1}^{t} a_{Pj}\hat{\varphi}_j \text{, at } p = 1,2,\ldots,t \text{,}$$

where $f^{(P)}$ - function of analysis of the system from the P-component; a_{Pj} - the jth characteristics of the Pth component. Equation (5) in a mathematical registration can be presented as:

$F = A\hat{\Phi}$,

where $F = \{f^{(1)}, f^{(2)}, \ldots, f^{(t)}\}$; $\Phi = \{\hat{\varphi}_1, \hat{\varphi}_2 \ldots, \hat{\varphi}_t\}$; $A = \lfloor a_{pj} \rfloor$

The components can be received like the solution of the relative equation

(6)
$$(R - \gamma I)\vec{a} = 0 \text{,}$$

where R is a matrix of coefficient of correlation for the variable quantities $\varphi_1, \varphi_2, \ldots, \varphi_t$; $\vec{a}$ - orthostandardized own vector of the linear transformation of R; γ -corresponding of R definite number; I-single matrix. Every negative number γ is root, characterizing the equation

(7)
$$(R - \gamma I) = 0 \text{,}$$

Taking the real values of the vector λ and substituting it in (6), a_{1j} is defined in equation (5) for the first component $f^{(1)}$. The same goes for a_2, a_3 for $f^{(2)}$, $f^{(3)}$, etc. The values for γ dispersion of the basic components and the sum of the dispersions $\gamma_1 + \gamma_2 + \ldots\gamma_t$ is equal to the sum of the dispersions $\left(\sigma^2_{\varphi_1} + \sigma^2_{\varphi_2} + \ldots\sigma^2_{\varphi_t}\right)$ of the initial characteristics of the system.

It can be seen from equation (5) that with the change of $f^{(p)}$, the most important are the characteristics of φ_i which provide the biggest value of a_{pj}. This means that if we analyze tha value of a_{pj}, we can choose such characteristics that could influence the change in the greatest extend of the basic components. This is a possibility to specify a set of the largest number of independent characteristics when designing optical systems. The sign in front of a_{pj} in equation (5) show a positive or negative link with φ_i in a specific component, which allows comparing the aberration image, which is defined by the basic components, to the requirements. On the basis of the examination of a_{pj} of the basic components, we can do a content analysis, i.e. the main components can be seen as basic parameters of the optic system and they present important regularities when forming the quality of the image.

In the analysis of the common factors, the basic equations are written in a matrix mode:

(8)
$$E = QF + U \text{,}$$

where $Q = (q_{pj})$ - rectangular matrix with sizes t x v for the coefficient of linear transformation;
U - transpose matrix;
$E = (l_j) - t$ metric vector with characteristic $F = (f^{(p)})$.

When factor analysis is used, there is a link between the t correlated characteristics φ_i and $\nu(\nu < t)$, when calculating the matrix Q, the coefficient of linear transformation is q_p when the correlation matrix R is known. We can suppose that U does not depend on F and no U_i is correlated with each other, i.e. matrix $V = M(UU^T)$ has a diagonal mode, where M is an operator of the mathematical expectation; U – transpose matrix. So, equation 8 with the help of the operator for mathematical expectation M is transformed into

$$R = QQ^T + M$$

where Q^T is a transpose matrix of Q

Because the coefficients q_{pj} are unknown in the factor model, the characteristics $\varphi_1, \varphi_2, ..., \varphi_i$ are approximated with the help of linear functions. The approximation of the characteristics φ_i is done with the help of $f^{(1)}, f^{(1)}, ..., f^{(\nu)}$.

In conclusion, we can say that the presented model of factor analysis ensures the creation of an algorithm of the designing process of complex optic systems. It is very necessary when designing photometric and spectrophotometric optic-electronic devices. These and similar optic systems require a significant extend of chromatic aberration removal to ensure good quality at the exit of the system. The authors of the paper realize that the presented topic is very complex, diverse and voluminous and that it cannot be completely solved but they also consider that they have contributed to its mastering.

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